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CHARTS FOR CHECKING THE STABILITY OF COMPRESSION MEMBERS IN TRUSSES

By K. Borkmann

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CHARTS FOR CHECKING THE STABILITY OF COMPRESSION MEMBERS IN TRUSSES*

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SUMMARY

The present report contains a set of charts developed for computing the fixity effect on a compression member in a truss through its adjacent members, the amount of fixity being considered variable with the particular total truss load. The use of the charts is illustrated on two- and three-bay systems, as well as on a triangular truss.

I. INTRODUCTION

In practice, the more general method of checking the stability of trusses, as of fuselages or wing spars, for instance, heretofore, has been to take the effect of fixity into account through fixity factors introduced as constants.

But this way of appraising the effect may lead to erroneous results, because the amount of fixity does not merely hinge upon the truss design but largely also on the stress existing at the moment in the whole system. The usual procedure of checking the buckling strength of a truss within its plane of the system with spatially defined nodal points under a certain stress, is to resolve the entire truss into groups of members, the members of each group being assumed rigid, but the individual groups as being hinged. (The result is then still somewhat on the safer side.)

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*"Kurventafeln für den Stabilitätsnachweis ebener Stabgruppen." Luftfahrtforschung, vol. 13, no. 1, January 20, 1936, pp. 1-9.

The buckling criterion (within the plane of the system) of such an individual group with, say, m-rigid nodal points, is the first occurrence of zero in the denominator determinant of an m-term system of equations, wherein the nodal point rotations represent the unknown factors, while the coefficients show the constants EJ/l of the individual members as well as trigonometric functions of the (varying with the amount of stress in member) instability criteria $\alpha = l \sqrt{\frac{S}{EJ}}$ (again of the individual members) (reference 1).

To adduce such a stability proof is very tedious because the coefficients must be successively established for different multiples of the given loading conditions with predetermined arrangement and loading, the value of the denominator determinant computed therefrom in order to find at last by interpolation that load stage at which this determinant becomes zero; that is, the individual group buckles. This expenditure of time is particularly undesirable when, during the designing, several dimensions have to be compared.

To forego this paper work, the following charts, based on the same formulas, have been developed and which, once the values of $\frac{EJ}{l}$ and $\alpha = l \sqrt{\frac{S}{EJ}}$ have been determined, disclose whether or not the individual group is buckling resistant (within its plane of the system).

II. GENERAL REMARKS ON THE USE OF THE CHARTS

1. Applicability to Buckling beyond Limit of Proportionality

The charts can be used equally well when one or more members of an individual group under predetermined loading have already exceeded the proportionality limit σ_P, provided these superelastically stressed members are computed with Young's modulus (τE) suitably reduced to conform with its stress σ, rather than with E itself.

According to a previously advanced suggestion (reference 2), it is expedient to estimate τ in such a way as to afford a linear interpolation of the particular σ values between $\tau = 1$ and $\tau = 0$ for σ_P and $\sigma_{0.2}$ (fig. 1).

2. Notation

The identifying value of a compression member of length l, bending stiffness EJ, and axial force S in the buckling proof, is the

$$\underline{\text{Instability criterion} \quad \alpha = l\sqrt{\frac{S}{EJ}}}$$

For practical reasons, the charts show the quantity $\left(\frac{\alpha}{\pi}\right)^2 = \frac{S}{S_K}$ (S_K = "natural buckling load" of the member with both ends hinged) rather than α.

If, in a group of members at a given stress, each individual α is smaller than π, that is, $\left(\frac{\alpha}{\pi}\right)^2$ less than 1, then, of course, each member of itself is buckling resistant (with fixed nodal points) and so also is the whole group.

If any individual α is greater than π, that is, $\left(\frac{\alpha}{\pi}\right)^2$ greater than 1, then the pertinent member with elastic support at both ends, would of itself buckle. It therefore needs to be proved that this so-called "buckling member" forms an individual group with one or more of its adjacent members, and which - assumedly hinged to the rest of the truss - is resistant to buckling in its plane. Every not completely utilized compression or tension member may serve as adjacent member. For a tension member $\alpha = l\sqrt{\frac{Z}{EJ}}$, where Z = axial force.

The use of the charts further requires the

$$\underline{\text{Bar parameter} \quad \varphi_N = \left(\frac{EJ}{l}\right)_N : \left(\frac{EJ}{l}\right)_K}$$

for each adjacent member within the individual group with "buckling member" K and the "adjacent members" N (N = 1, 2 .. i).

The terms "buckling" and "adjacent" member within an individual group are chosen merely with a view to better illustration. Admittedly, upon reaching the buckling limit, the individual group buckles as a whole, not the buckling strut alone, within the individual group.

If the truss system has several such buckling struts, then a single group in itself buckling resistant must be found for each of these buckling members whereby, of course, a so-called adjacent member may not belong more than once to any single group.

As previously pointed out, the statements hereinafter as regards buckling within the plane of the elastic system, apply to groups of members with spatially defined (although rotatable) nodal points. Further, it is presumed that the principal axes of the members run parallel or at right angles to the plane of the elastic system.

III. APPLICATION OF CHARTS

Figure 2 illustrates the various single groups which are to be checked as regards buckling strength on the charts.

<u>a) Buckling of a two-bay group of members in its plane</u>. (Chart A, system of arrangement at lower left).- Given:

$$\varphi_N = \left(\frac{EJ}{l}\right)_N : \left(\frac{EJ}{l}\right)_K,$$

$$\left(\frac{\alpha_K}{\pi}\right)^2 > 1,$$

$$\left(\frac{\alpha_N}{\pi}\right)^2 < 1,$$

if the adjacent member is a not fully utilized compression member.

If, on the other hand, the adjacent member is a tension member with axial force Z, then $\alpha_N = l \sqrt{\frac{Z}{EJ}}$, in which case $\left(\frac{\alpha_N}{\pi}\right)^2$ may be $\lesseqgtr 1$, and the part of the chart marked "tension member" should be used for $\left(\frac{\alpha_N}{\pi}\right)^2$.

Then find the point A (see, for example, fig. 4),

having the coordinates $\left(\frac{\alpha_N}{\pi}\right)^2$ and $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_K}{\pi}\right)^2$. If A lies below the curve with the given parameter φ_N, then the group of members is resistant to buckling, and vice versa.

Aside from that, chart A also affords - though for the elastic range of both members only - the extent of existing buckling strength:

Connect point A with the polar P: $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$; $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 0$ and draw the line PA with the curve corresponding to the given φ_N so as to intersect S. Then $j = PS/PA$ is the buckling strength.

Note.- The charts are based on the following formulas:

General:

$$u = \frac{\alpha^2}{1 - \frac{\alpha}{\tan\alpha}} \qquad v = \frac{\alpha^2\left(1 - \frac{\alpha}{\tan\alpha}\right)}{\left(1 - \frac{\alpha}{\tan\alpha}\right)^2 - \left(1 - \frac{\alpha}{\sin\alpha}\right)^2}$$

$$w = \frac{\alpha^2\left(1 - \frac{\alpha}{\sin\alpha}\right)}{\left(1 - \frac{\alpha}{\tan\alpha}\right)^2 - \left(1 - \frac{\alpha}{\sin\alpha}\right)^2}$$

Chart A:

$$u_{K_r} + u_N\,\varphi_N = 0$$

Chart B:

$$(u_{K_r}')_l\,(u_{K_r}')_r - v_{K_r}\,[(u_{K_r}')_l + (u_{K_r}')_r] + u_{K_r}\,v_{K_r} = 0$$

Chart C:

$$\varphi_1\,v_1\,\varphi_2^2\,(uv)_2 + \varphi_2\,v_2\,\varphi_1^2\,(uv)_1$$
$$+ v_{K_r}\,[(\varphi_1\,v_1 + \varphi_2\,v_2)^2 - (\varphi_1^2\,w_1^2 + \varphi_2^2\,w_2^2)]$$
$$+ (uv)_{K_r}\,(\varphi_1\,v_1 + \varphi_2\,v_2) - 2w_{K_r}\,\varphi_1\,w_1\,\varphi_2\,w_2 = 0$$

Special Case of Two-Bay System

a) The values given for the lengths, stiffness, and axial forces are merely those for mutual comparison of

$$\frac{l_K}{l_N}, \quad \frac{(EJ)_K}{(EJ)_N} \quad \text{and} \quad \frac{S_K}{S_N}$$

rather than their absolute values.

Even so, the buckling limit of the individual group may be taken from chart A (solely for the case of elastic stress in both members) by means of the auxiliary straight line for $\tan \beta$, located between chart and pole P.

First determine:

the parameter $$\varphi_N = \frac{(EJ)_N}{(EJ)_K} \frac{l_K}{l_N}$$

and

the auxiliary value $$\tan \beta = \frac{l_N^2}{l_K^2} \frac{S_N}{S_K} \frac{(EJ)_K}{(EJ)_N} = \left(\frac{\alpha_N}{\alpha_K}\right)^2$$

From the given comparative values, ascertain point B on the auxiliary straight line for the obtained value $\tan \beta$, then extend PB over B until it intersects the curve (point C) defined by parameter φ_N. The ordinate $(\alpha_{K_r}/\pi)^2$ of point C is a criterion for the fixity effect of buckling strut K through the adjacent member N, since the individual group does not buckle until the stress in the buckling strut has become $(\alpha_{K_r}/\pi)^2$ times its "natural" buckling load $S_K = \dfrac{(EJ)_K \, \pi^2}{l_K^2}$.

b) Fixity of a "buckling" strut at one extremity through several "neighboring" struts (chart A, system according to fig. 25).

Given data:

$$\left(\frac{\alpha_K}{\pi}\right)^2 > 1, \quad \text{for buckling strut}$$

(N = 1, 2 ... i, for each neighboring strut*

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*If one or more of these are tension members, the statement under III,a $(\alpha_N/\pi)^2$ is applicable.

$$\left(\frac{\alpha_N}{\pi}\right) < 1 \quad \text{and} \quad \varphi_N = \left(\frac{EJ}{l}\right)_N : \left(\frac{EJ}{l}\right)_K$$

First ascertain whether or not an isolated adjacent member already forms a buckling-resistant single group of itself with the buckling member. This is attempted according to III,a.

If such is not the case, find the reduced bar parameter $\overline{\varphi_N}$ referred to $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ for each of the adjacent members N on the chart as follows:

Proceeding upward from the given abscissa $\left(\frac{\alpha_N}{\pi}\right)^2$, continue until point D_N meets the curve corresponding to the pertinent parameter φ_N, then pass horizontally over it to the axis $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$, and determine on it the relative $\overline{\varphi_N}$. (See fig. 4.)

Then if the point $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$; $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_K}{\pi}\right)^2$ lies below the curve with the parameter $\overline{\varphi} = \Sigma\,\overline{\varphi_N}$, the individual group is resistant to buckling and, if above the curve with parameter $\overline{\varphi} = \Sigma\,\overline{\varphi_N}$, the individual group is not safe against buckling.

With a great number of adjacent members, it is possible that two or three of them already afford a sufficiently high value of $\overline{\varphi} = \Sigma\,\overline{\varphi_N}$. (The introduction of reduced parameter $\overline{\varphi_N}$ in the sense of an intermediate calculation, does not signify omission of the axial stresses in the adjacent members. For the case of only one adjacent member, the procedure is as that described under III,a.)

c) <u>Buckling of a three-bay group in its plane</u> (charts A, B; system illustrated below chart B).

Given: $\varphi_{N_l} = \left(\frac{EJ}{l}\right)_{N_l} : \left(\frac{EJ}{l}\right)_K$

$$\varphi_{N_r} = \left(\frac{EJ}{l}\right)_{N_r} : \left(\frac{EJ}{l}\right)_K$$

$$\left(\frac{\alpha_K}{\pi}\right)^2 > 1,$$

$$\left(\frac{\alpha_{N_l}}{\pi}\right)^2 < 1^* \quad \text{and} \quad \left(\frac{\alpha_{N_r}}{\pi}\right)^2 < 1^*$$

Assuming a joint between compression member and the right adjacent member N_r say, check whether the individual group (III,a) consisting of member K and the left adjacent member N_l is in itself buckling proof. If not, assume a joint between K and N_l and check whether the group consisting of K and N_r is in itself buckling proof. If even this is not, then take the

$$\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_{K_r}'}{\pi}\right)_l^2$$

value relative to $\left(\frac{\alpha_{N_l}}{\pi}\right)^2$ and to the curve with parameter φ_{N_l} (or the $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_{K_r}'}{\pi}\right)_r^2$ relative to $\left(\frac{\alpha_{K_r}}{\pi}\right)^2$ and to the curve with parameter φ_{N_r} from chart A.

From $\left(\frac{\alpha_{K_r}'}{\pi}\right)_l^2$ and $\left(\frac{\alpha_{K_r}}{\pi}\right)_r^2$ then determine the value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2$ according to chart B, which corresponds to the buckling load of member K for the given size of member and to the stress in the two adjacent members. If the $\left(\frac{\alpha_{K_r}}{\pi}\right)^2$ taken from the chart exceeds the given $\left(\frac{\alpha_K}{\pi}\right)^2$, then the three-bay group is secure from buckling.

d) <u>Fixity of a buckling strut through several adjacent members at its two ends</u> (Charts A and B, group arrangement according to fig. 2d).- Given: $(\alpha_K/\pi)^2 > 1$ for buckling member, for each adjacent member (see footnote, p. 6) applied on the left end of the buckling member: $N_l = 1_l, 2_l \ldots i_l$)

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*See footnote, p. 6.

$$\left(\frac{\alpha_{N_l}}{\pi}\right)^2 < 1 \quad \text{and} \quad \varphi_{N_l} = \left(\frac{EJ}{l}\right)_{N_l} : \left(\frac{EJ}{l}\right)_K$$

and for every adjacent member applied on the right end of the buckling member (see footnote, p. 6) ($N_r = 1_r, 2_r \dots i_r$)

$$\left(\frac{\alpha_{N_r}}{\pi}\right)^2 < 1 \quad \text{and} \quad \varphi_{N_r} = \left(\frac{EJ}{l}\right)_{N_r} : \left(\frac{EJ}{l}\right)_K$$

Assume a joint - say, at the right end of K, and verify whether the individual group formed of K and all adjacent members N_l (applied at left end of K) is by itself safe against buckling. If not, apply the same procedure to the other (left) end of K; if this fails also, take the value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_{K_r}'}{\pi}\right)_l^2$ for $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ and the curve with parameter $\overline{\varphi}_l = \Sigma\, \overline{\varphi}_{N_l}$ or the value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_{K_r}'}{\pi}\right)_r^2$ belonging to $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ and the curve with $\overline{\varphi}_r = \Sigma\, \overline{\varphi}_{N_r}$ from chart A. Naturally, every one of these values is smaller than the given value $\left(\frac{\alpha_K}{\pi}\right)^2$.

Following the determination of $\left(\frac{\alpha_{K_r}'}{\pi}\right)_l^2$ and $\left(\frac{\alpha_{K_r}'}{\pi}\right)_r^2$ the rest of the procedure is as before.

e) <u>Buckling of a triangular truss within its plane of the system</u> (Chart C, arrangement according to figure 2e).- As a rule, a member which (pinned support at both ends), with a given loading, precisely reaches its natural buckling load, i.e., for which $(\alpha/\pi)^2 = 1$, may not be utilized for fixation of an adjacent buckling member with the same loading condition, since its ends are free to turn without offering resistance. By this argument an open multi-bay system of members is in one of its buckling conditions when, precisely, $(\alpha_i/\pi)^2 = 1$ for each of its individual members i (fig. 3a).

But the conditions are different in the triangular truss. Since, on buckling of such a truss at least one member must buckle in an S shape (fig. 3b), it is very

well possible that $(\alpha/\pi)^2$ is already greater than 1 for all three members, while the triangle itself has still not buckled within its plane.

In cases where for two, or even all three members $\left(\frac{\alpha}{\pi}\right)^2 \geqq 1$, the use of the charts proceeds with any designated member as "buckling strut" (member 3), and the other two as "adjacent members" (members 1 and 2).

For the proof of the buckling of a triangular truss, i.e., the determination of value $(\alpha_{K_r}/\pi)^2$ for the actual buckling status, four individual factors are available, each of which is able to exert a marked effect on the result: the parameters φ_1 and φ_2, and the instability criteria $(\alpha_1/\pi)^2$ and $(\alpha_2/\pi)^2$ of the adjacent members. For this reason, it is impossible to include every existing case exact in a few charts.

And so chart C comprises a series of individual charts for the six most essential combinations of instability criteria of members 1 and 2, from which the actual buckling load of member 3 (buckling strut K) may be read in function of the two parameters $\varphi_1 = \left(\frac{EJ}{l}\right)_1 : \left(\frac{EJ}{l}\right)_3$ and $\varphi_2 = \left(\frac{EJ}{l}\right)_2 : \left(\frac{EJ}{l}\right)_3$ in form of value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2$ for these individual cases.

For the actually existing instability criteria $\left(\frac{\alpha_1}{\pi}\right)^2$ and $\left(\frac{\alpha_2}{\pi}\right)^2$ the particular value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2$ can then be estimated by interpolation of the results of the contiguous special cases.

If then, finally, the obtained value of $\left(\frac{\alpha_{K_r}}{\pi}\right)^2$ exceeds the given $\left(\frac{\alpha_K}{\pi}\right)^2$ of member 3, the triangular system is resistant to buckling.

The choice of these combinations of $\left(\frac{\alpha_1}{\pi}\right)^2$ and $\left(\frac{\alpha_2}{\pi}\right)^2$ in the form of

$\left(\frac{\alpha_1}{\pi}\right)^2 =$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	1	1
$\left(\frac{\alpha_2}{\pi}\right)^2 =$	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$

was based upon the following considerations:

1. According to experience, it does not affect the result of a buckling calculation very much when a member under tension is introduced as unstressed. (The result still leaves one on the safer side.) Whence the introduction of $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ as lower limiting case.

2. Although we had previously stated that theoretically the $(\alpha/\pi)^2$ values of all three members may exceed 1, without causing buckling - in practice, however, the adjacent members will not be fully utilized as regards buckling, i.e., $\left(\frac{\alpha_N}{\pi}\right)^2 < 1$, so that $\left(\frac{\alpha_N}{\pi}\right)^2 = 1$ actually represents a rare and unfavorable case.

For occasionally encountered special cases with still higher instability criteria of adjacent members, a separate proof of the buckling strength must be adduced (determination of zero value of determinant of denominator).

3. Between these two limit values 0 and 1, the mean value $\frac{1}{2}$, serves as basis of these charts.

IV. EXAMPLES

To III,a: Buckling of a two-bay system (arrangement, fig. 2a).- What is the buckling safety of the two-bay system K-N with the given values?

	K			N		
	EJ	l	S	EJ	l	S
	kg cm^2	cm	kg	kg cm^2	cm	kg
1. Example	3×10^6	50	-15,000	3.5×16^6	40	-8,000
2. Example	3×10^6	45	-18,000	2.5×16^6	50	+3,000

Example 1

$$\varphi_N = \left(\frac{EJ}{l}\right)_N : \left(\frac{EJ}{l}\right)_K = \left(\frac{3.5\times10^6}{40}\right) : \left(\frac{3\times10^6}{50}\right) = 1.46$$

$$\alpha_K = \left(l\sqrt{\frac{S}{EJ}}\right)_K = 50\sqrt{\frac{15000}{3\times10^6}} = 3.54$$

$$\left(\frac{\alpha_K}{\pi}\right)^2 = 1.27$$

$$\alpha_N = \left(l\sqrt{\frac{S}{EJ}}\right)_N = 40\sqrt{\frac{8000}{3.5\times10^6}} = 1.91$$

$$\left(\frac{\alpha_N}{\pi}\right)^2 = 0.37$$ (not fully utilized compression member).

The system is resistant to buckling because, according to chart A (fig. 4):

$$j = \frac{PS}{PA} = 1.1$$

Example 2

$$\varphi_N = \left(\frac{2.5\times10^6}{50}\right) : \left(\frac{3\times10^6}{45}\right) = 0.75$$

$$\alpha_K = 45\sqrt{\frac{18000}{3\times10^6}} = 3.48; \quad \left(\frac{\alpha_K}{\pi}\right)^2 = 1.23$$

$$\alpha_N = 50\sqrt{\frac{3000}{2.5\times10^6}} = 1.73; \quad \left(\frac{\alpha_N}{\pi}\right)^2 = 0.30$$ (tension member).

The system is resistant to buckling because (fig. 4):

$$j = \frac{PS}{PA} = 1.1$$

<u>To III,a: Special case</u>.- A continuous member of bending stiffness EJ and under axial (compressive) load S is braced in one of its third points against buckling in the plane. What is the buckling load of the member when

pin-jointed at both ends? Total length l.

It is:

$$l_K = \frac{2}{3} l; \quad l_N = \frac{1}{3} l$$

Further:

$$\frac{(EJ)_K}{(EJ)_N} = 1; \quad \frac{S_K}{S_N} = 1; \quad \left(\frac{l_K}{l_N}\right) = 2$$

Then:

$$\varphi_N = \frac{(EJ)_N}{(EJ)_K} \left(\frac{l_K}{l_N}\right) = 2$$

$$\tan \beta = \frac{l_N^2}{l_K^2} \frac{S_N}{S_K} \frac{(EJ)_K}{(EJ)_N} = \frac{1}{4} = 0.25$$

for which, according to chart A:

$$\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 1.51.$$

Finally,

$$S_{K_r} = 1.51 \frac{(EJ)_K \pi^2}{l_K^2}$$

$$= \frac{1.51}{4/9} \frac{E J \pi^2}{l^2} = 3.4 \frac{E J \pi^2}{l^2}$$

A support at one of the three points raises the buckling load of the whole member by 3.4 times of its original figure.

<u>To III,b: Fixation of a "buckling strut" through several "adjacent members" at one end</u> (arrangement as in fig. 2b) (i = 3).- Given:

Member	EJ	l	S
	kg cm²	cm	kg
K	4×10^6	60	-15,000
1	4×10^6	50	-13,000
2	1.8×10^6	70	-2,000
3	1.5×10^6	80	+5,000

It is:

$$\alpha_K = 60\sqrt{\frac{15000}{4\times10^6}} = 3.67; \quad \left(\frac{\alpha_K}{\pi}\right)^2 = 1.37$$

$$\varphi_1 = \left(\frac{4\times10^6}{50}\right) : \left(\frac{4\times10^6}{60}\right) = 1.2$$

$$\alpha_1 = 50\sqrt{\frac{13000}{4\times10^6}} = 2.85; \quad \left(\frac{\alpha_1}{\pi}\right)^2 = 0.825$$ (compression member not fully utilized).

K and member 1 alone are not resistant to buckling (their buckling strength being only $j = 0.9$).

$\overline{\varphi}_1 = 0.3$ (See fig. 4.)

$$\varphi_2 = \left(\frac{1.8\times10^6}{70}\right) : \left(\frac{4\times10^6}{60}\right) = 0.386$$

$$\alpha_2 = 70\sqrt{\frac{2000}{1.8\times10^6}} = 2.33; \quad \left(\frac{\alpha_2}{\pi}\right)^2 = 0.55$$ (compression member not fully utilized).

K and member 2 alone are not resistant to buckling because $j = 0.8$.

$\overline{\varphi}_2 = 0.2$

$$\varphi_3 = \left(\frac{1.5\times10^6}{80}\right) : \left(\frac{4\times10^6}{60}\right) = 0.282$$

$$\alpha_3 = 80\sqrt{\frac{5000}{1.5\times10^6}} = 4.61; \quad \left(\frac{\alpha_3}{\pi}\right)^2 = 2.16$$ (tension member).

K and member 3 alone are not resistant to buckling in view of $j = 0.9$.

$\overline{\varphi}_3 = 0.53$

Altogether, it is: $\overline{\varphi} = 0.3 + 0.2 + 0.53 = 1.03$.

The system K, 1, 2, 3 is resistant to buckling because chart A shows the point with the coordinates $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ and $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = \left(\frac{\alpha_K}{\pi}\right)^2 = 1.37$ below the curve with the parameter $\overline{\varphi} = 1.03$. (It approximately lies on a curve with the parameter $\overline{\varphi} = 0.88$.)

To III,c: Buckling of a three-bay system in its plane (arrangement as of fig. 2c).- With given:

Member	EJ	l	S
	kg cm²	cm	kg
K	3.5×10^6	80	-15,000
N_L	3×10^6	40	-5,000
N_r	4×10^6	50	+8,000

It is:

$$\alpha_K = 80\sqrt{\frac{15000}{3.5\times10^6}} = 5.24;\quad \left(\frac{\alpha_K}{\pi}\right)^2 = 2.78$$

$$\varphi_{N_l} = \left(\frac{3\times10^6}{40}\right) : \left(\frac{3.5\times10^6}{80}\right) = 1.715$$

$$\alpha_{N_l} = 40\sqrt{\frac{5000}{3\times10^6}} = 1.63;\quad \left(\frac{\alpha_{N_l}}{\pi}\right)^2 = 0.27$$ (compression member not fully utilized)

K and N_l alone are not resistant to buckling (even when rigidly clamped at one end only) $(\alpha_{K_r}/\pi)^2$ can only amount to 2.0457* in the most favorable case).

$^*\left(\frac{\alpha}{\pi}\right)^2 = 2.0457$ corresponds to the condition: $\alpha = \tan\alpha$ (partially rigid restraint).

$$\left(\frac{\alpha_{K_r}{}'}{\pi}\right)_l^2 = 1.50 \quad \text{(according to chart A).}$$

$$\varphi_{N_r} = \left(\frac{4 \times 10^6}{50}\right) : \left(\frac{3.5 \times 10^6}{80}\right) = 1.83$$

$$\alpha_{N_r} = 50\sqrt{\frac{8000}{4 \times 10^6}} = 2.24; \ \left(\frac{\alpha_{N_r}}{\pi}\right)^2 = 0.51 \quad \text{(tension member)}$$

K and N_r alone are not resistant to buckling (see above).

$$\left(\frac{\alpha_{K_r}{}'}{\pi}\right)_r^2 = 1.64 \quad \text{(according to chart A).}$$

Chart B gives for

$$\left(\frac{\alpha_{K_r}{}'}{\pi}\right)_l^2 = 1.50 \quad \text{and} \quad \left(\frac{\alpha_{K_r}{}'}{\pi}\right)_r^2 = 1.64 \quad \text{the value:}$$

$$\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.34.$$

This figure being smaller than the given $\left(\frac{\alpha_K}{\pi}\right)^2 = 2.78$, the system is not resistant to buckling.

<u>To III,d: Fixation of a "buckling strut" through several "adjacent members" on its two ends</u> (arrangement as of fig. 2d) ($i_l = 2$; $i_r = 3$).- With given

Member	EJ	l	S
	kg cm^2	cm	kg
K	1.2×10^6	50	-10,000
1_l	1.0×10^6	50	-3,000
2_l	$.5 \times 10^6$	80	+1,800
1_r	1.4×10^6	50	-4,000
2_r	$.8 \times 10$	70	+2,000
3_r	1.5×10	30	-8,000

it is:

$$\alpha_K = 50\sqrt{\frac{10000}{1.2 \times 10^6}} = 4.56; \quad \left(\frac{\alpha_K}{\pi}\right)^2 = 2.11$$

$\left(\frac{\alpha_K}{\pi}\right)^2$ being greater than 2.0457, buckling strut K together with the adjacent members at only one end, cannot form an individual system which by itself is resistant to buckling.

$$\varphi_{1_l} = \left(\frac{1.0 \times 10^6}{50}\right) : \left(\frac{1.2 \times 10^6}{50}\right) = 0.83$$

$$\alpha_{1_l} = 50\sqrt{\frac{3000}{1.0 \times 10^6}} = 2.74; \quad \left(\frac{\alpha_{1_l}'}{\pi}\right)^2 = 0.76$$ (compression strut not fully utilized)

$$\overline{\varphi}_{1_l} = 0.28$$

$$\varphi_{2_l} = \left(\frac{0.5 \times 10^6}{80}\right) : \left(\frac{1.2 \times 10^6}{50}\right) = 0.26$$

$$\alpha_{2_l} = 80\sqrt{\frac{1800}{0.5 \times 10^6}} = 4.8; \quad \left(\frac{\alpha_{2_l}}{\pi}\right)^2 = 2.34$$ (tension strut)

$$\overline{\varphi}_{2_l} = 0.52$$

$$\overline{\varphi}_l = 0.28 + 0.52 = 0.80$$

For $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ and $\overline{\varphi}_l = 0.80$, chart A shows

$$\left(\frac{\alpha_{K_r}'}{\pi}\right)^2_l = \underline{1.35}.$$

$$\varphi_{1_r} = \left(\frac{1.4 \times 10^6}{50}\right) : \left(\frac{1.2 \times 10^6}{50}\right) = 1.17$$

$$\alpha_{1_r} = 50\sqrt{\frac{4000}{1.4 \times 10^6}} = 2.67; \quad \left(\frac{\alpha_{1_r}}{\pi}\right)^2 = 0.72$$ (compression strut not fully utilized)

$$\overline{\varphi}_{1_r} = 0.46$$

$$\varphi_{2_r} = \left(\frac{0.8 \times 10^6}{70}\right) : \left(\frac{1.2 \times 10^6}{50}\right) = 0.476$$

$$\alpha_{2r} = 70\sqrt{\frac{2000}{0.8 \times 10^6}} = 3.5; \left(\frac{\alpha_{2r}}{\pi}\right)^2 = 1.24 \text{ (tension strut)}$$

$$\overline{\varphi}_{2_r} = 0.76$$

$$\varphi_{3_r} = \left(\frac{1.5 \times 10^6}{30}\right) : \left(\frac{1.2 \times 10^6}{50}\right) = 2.08$$

$$\alpha_{3_r} = 30\sqrt{\frac{8000}{1.5 \times 10^6}} = 2.19; \left(\frac{\alpha_{3_r}}{\pi}\right)^2 = 0.49 \text{ (compression strut not fully utilized)}$$

$$\overline{\varphi}_{3_r} = 1.32$$

$$\overline{\varphi}_r = 0.46 + 0.76 + 1.32 = 2.54$$

For $\left(\frac{\alpha_N}{\pi}\right)^2 = 0$ and $\overline{\varphi}_r = 2.54$, chart A gives

$$\left(\frac{\alpha_{K_r}'}{\pi}\right)_r^2 = \underline{1.65}$$

Chart B gives for

$$\left(\frac{\alpha_{K_r}'}{\pi}\right)_l^2 = 1.35 \quad \text{and} \quad \left(\frac{\alpha_{K_r}'}{\pi}\right)_r^2 = 1.65 \quad \text{the value:}$$

$$\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.15.$$

This figure being higher than the given $\left(\frac{\alpha_K}{\pi}\right)^2 = 2.11$; the system is resistant to buckling.

<u>To III,e: Buckling of a triangular truss within the plane of the truss</u> (arrangement as of fig. 2e).- Given

	1			2			3 = K		
	EJ	l	S	EJ	l	S	EJ	l	S
	kg cm^2	cm	kg	kg cm^2	cm	kg	kgcm^2	cm	kg
Example 1	3×10^6	55	-8500	2.5×10^6	65	-2000	2×10^6	60	-10000
Example 2	2×10^6	45	-1000	1×10^6	50	+8000	4×10^6	70	-20000

Example 1

$$\alpha_K = 60\sqrt{\frac{10000}{2\times10^6}} = 4.24; \quad \left(\frac{\alpha_K}{\pi}\right)^2 = 1.82$$

$$\varphi_1 = \left(\frac{3\times10^6}{55}\right) : \left(\frac{2\times10^6}{60}\right) = 1.64$$

$$\alpha_1 = 55\sqrt{\frac{8500}{3\times10^6}} = 2.93; \quad \left(\frac{\alpha_1}{\pi}\right)^2 = 0.87$$ (compression strut not fully satisfied)

$$\varphi_2 = \left(\frac{2.5\times10^6}{65}\right) : \left(\frac{2\times10^6}{60}\right) = 1.15$$

$$\alpha_2 = 65\sqrt{\frac{2000}{2.5\times10^6}} = 1.84; \quad \left(\frac{\alpha_2}{\pi}\right)^2 = 0.34$$ (compression strut not fully satisfied)

$\left(\frac{\alpha_1}{\pi}\right)^2 = 0.87$ lies between $\frac{1}{2}$ and 1

$\left(\frac{\alpha}{\pi}\right)^2 = 0.34$ lies between 0 and $\frac{1}{2}$

Chart C shows for the given values of $\varphi_1 = 1.64$ and $\varphi_2 = 1.15$

1 for $\left(\frac{\alpha_1}{\pi}\right)^2 = \frac{1}{2}$; $\left(\frac{\alpha_2}{\alpha}\right)^2 = 0$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.26$

2 for $\left(\frac{\alpha_1}{\pi}\right)^2 = \frac{1}{2}$; $\left(\frac{\alpha_2}{\pi}\right)^2 = \frac{1}{2}$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.22$

3 for $\left(\frac{\alpha_1}{\pi}\right)^2 = 1$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.11$

4 for $\left(\frac{\alpha_1}{\pi}\right)^2 = 1$; $\left(\frac{\alpha_2}{\pi}\right)^2 = \frac{1}{2}$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.06$

so that straight interpolation gives between (1) (2)

(5) for $\left(\frac{\alpha_1}{\pi}\right)^2 = \frac{1}{2}$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0.34$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.23$

between (3) (4)

(6) for $\left(\frac{\alpha_1}{\pi}\right)^2 = 1$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0.34$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.08$

and straight interpolation between (5) (6)

for $\left(\frac{\alpha_1}{\pi}\right)^2 = 0.87$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0.34$ a value $\left(\frac{\alpha_{K_r}}{\pi}\right)^2 = 2.12$

(The exact figure is 2.14 instead of 2.12.)

As this figure, 2.12 or 2.14, is greater than the given 1.82 of member 3 = K, the triangular truss is resistant to buckling (within its plane).

Example 2

$$\alpha_K = 70\sqrt{\frac{20000}{4\times10^6}} = 4.95; \quad \left(\frac{\alpha_K}{\pi}\right)^2 = 2.48$$

$$\varphi_1 = \left(\frac{2\times10^6}{45}\right) : \left(\frac{4\times10^6}{70}\right) = 0.78$$

$$\alpha_1 = 45\sqrt{\frac{1000}{2\times10^6}} = 1.01; \quad \left(\frac{\alpha_1}{\pi}\right)^2 = 0.10$$ (compression strut not fully used)

$$\varphi_2 = \left(\frac{4\times10^6}{50}\right) : \left(\frac{4\times10^6}{70}\right) = 1.4$$

$$\alpha_2 = 50\sqrt{\frac{8000}{4\times10^6}} = 2.24; \quad \left(\frac{\alpha_2}{\pi}\right)^2 = 0.51$$ (tension strut)

$\left(\frac{\alpha_1}{\pi}\right)^2 = 0.10$ lies between 0 and $\frac{1}{2}$

$\left(\frac{\alpha_2}{\pi}\right)^2 = 0.51$ (tension strut) is approximated at = 0

Chart 3 shows for $\varphi_1 = 0.78$ and $\varphi_2 = 1.4$

1 for $\left(\frac{\alpha_1}{\pi}\right)^2 = 0$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0$ a value $\left(\frac{\alpha_{Kr}}{\pi}\right)^2 = 2.14$

2 for $\left(\frac{\alpha_1}{\pi}\right)^2 = \frac{1}{2}$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0$ a value $\left(\frac{\alpha_{Kr}}{\pi}\right)^2 = 2.07$

Straight interpolation gives between 1 2

for $\left(\frac{\alpha_1}{\pi}\right)^2 = 0.10$; $\left(\frac{\alpha_2}{\pi}\right)^2 = 0$ a value $\left(\frac{\alpha_{Kr}}{\pi}\right)^2 = 2.13$

(The exact figure, with due allowance of $\left(\frac{\alpha_2}{\pi}\right)^2 = 0.51$ for the tension member, is 2.19 instead of 2.13).

This figure, 2.13 (or 2.19) being less than the given $\left(\frac{\alpha_K}{\pi}\right)^2 = 2.48$ for member 3 = K, the triangular system is not safe against buckling.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

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2. Borkmann, K.: Zur Berechnung der Stabilität von Stabgruppen bei Beanspruchung jenseits der Proportionalitätsgrenze. Z.F.M., vol. 24, no. 5, 1933, p. 139.

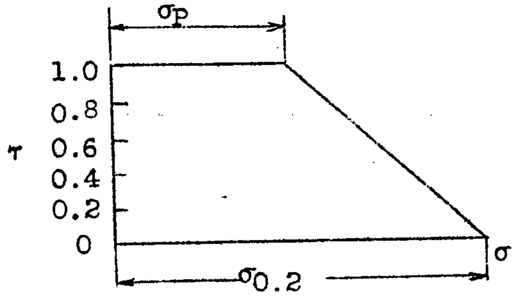


Fig.1= Reduction factor τ for the modulus of elasticity of outer-elastically stressed members (i.e. for $\sigma > \sigma_p$)

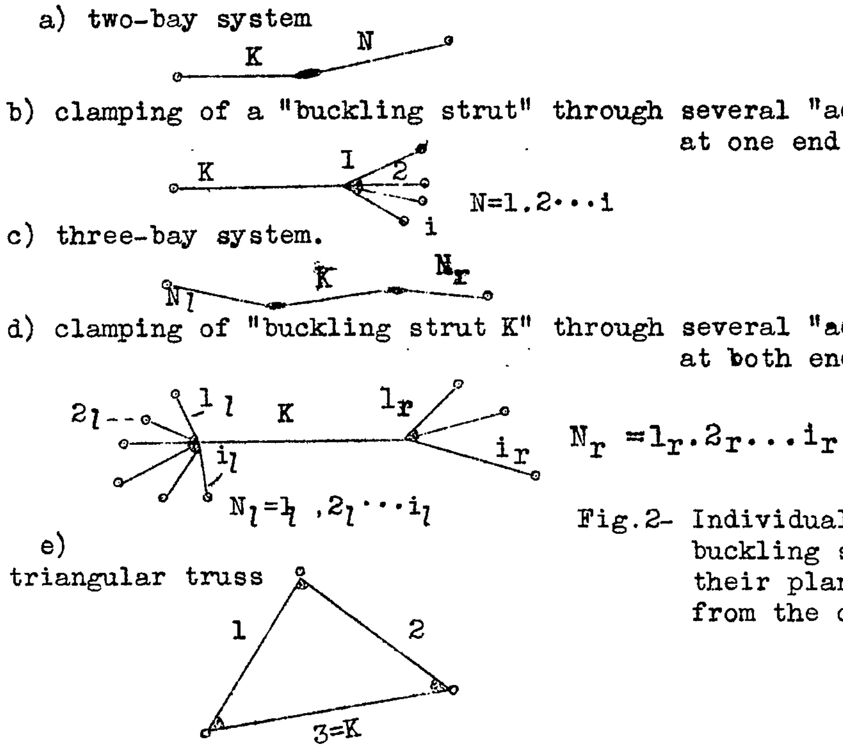


Fig.2- Individual groups whose buckling strength (within their planes) may be proved from the charts.

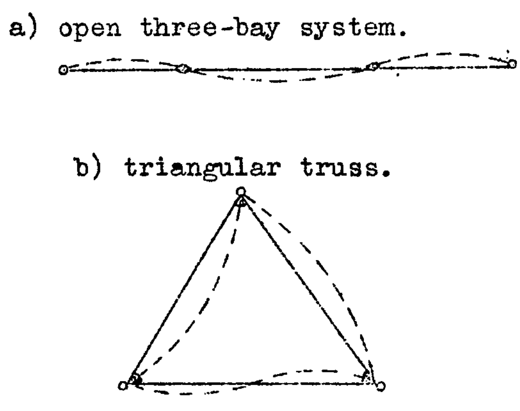
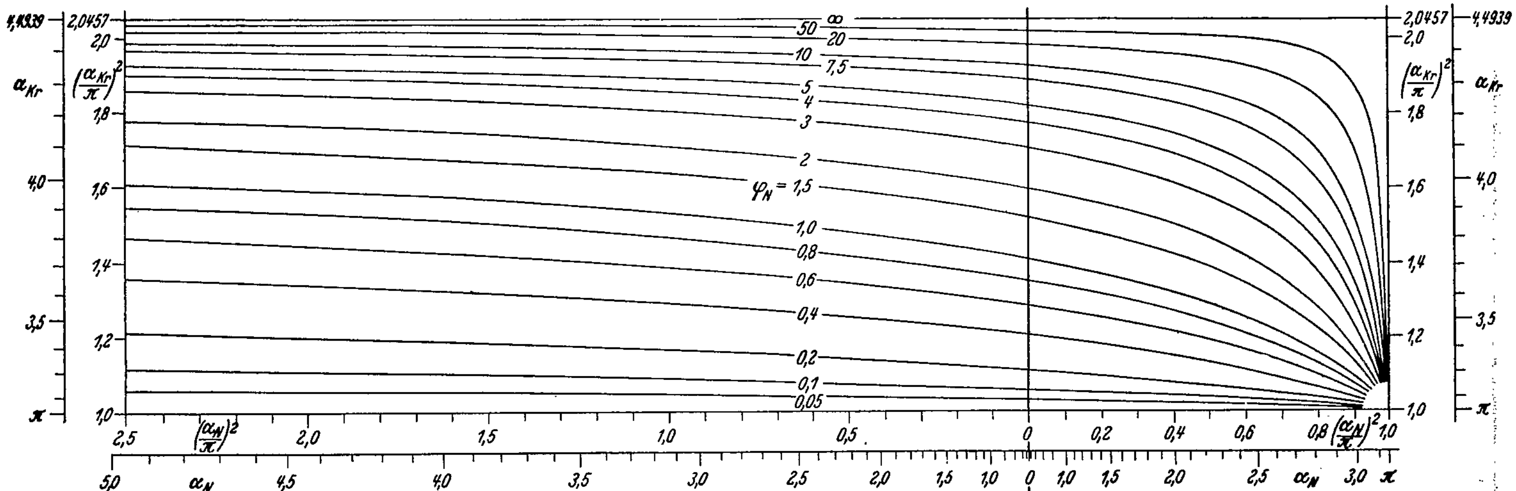
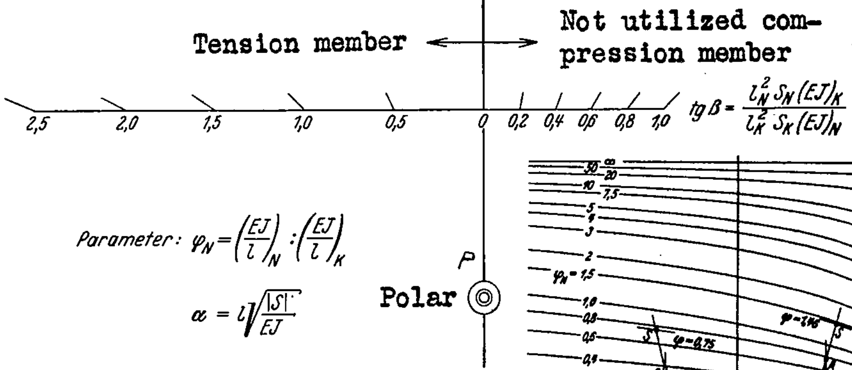


Fig.3- Buckling forms (on buckling within its plane of the system.)





System of arrangement

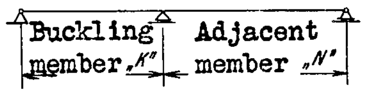
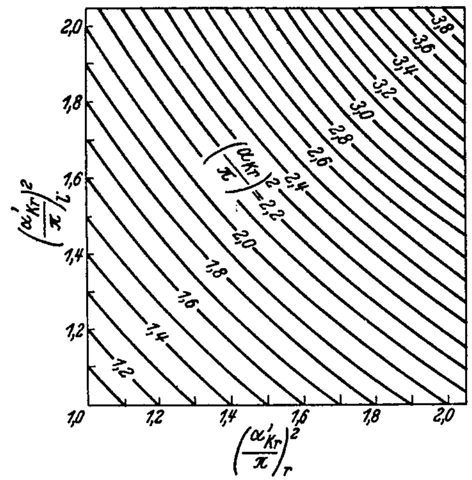


Chart A.- Buckling of a two-bay system in its plane.



Arrangement:

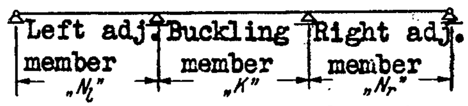


Chart B.- Buckling of a three-bay system in its plane.

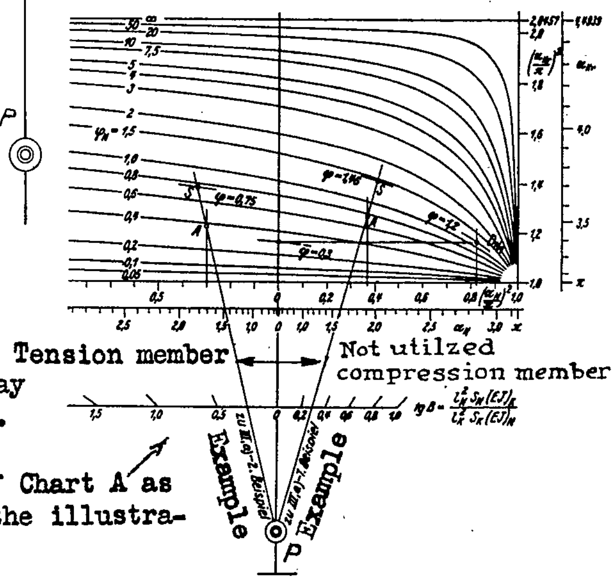
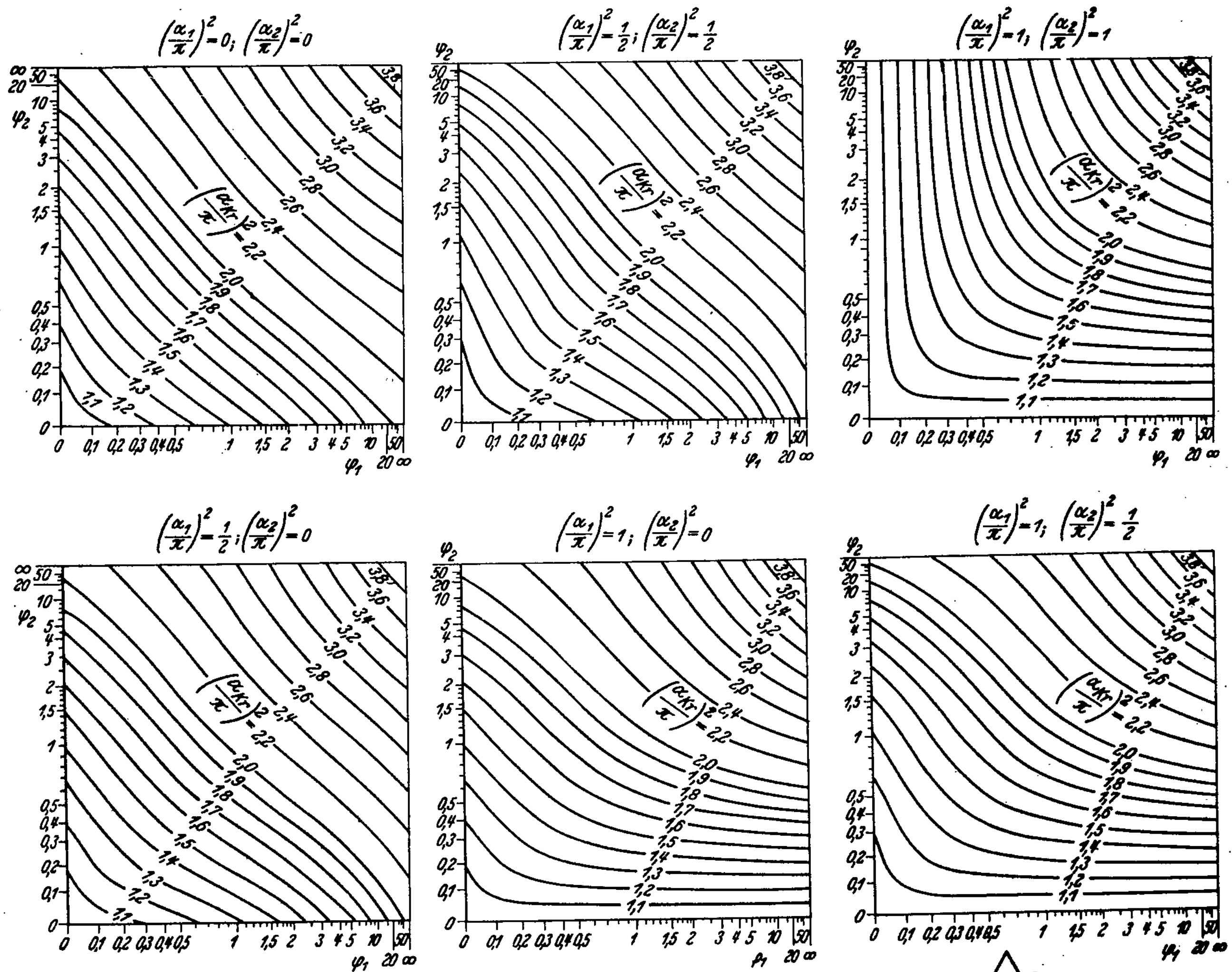


Figure 4.- Section of Chart A as used for the illustrative examples.



Es ist: $\varphi_1=\left(\frac{EJ}{l}\right)_1 : \left(\frac{EJ}{l}\right)_3$ $\quad \varphi_2=\left(\frac{EJ}{l}\right)_2 : \left(\frac{EJ}{l}\right)_3$ $\quad \alpha=l\sqrt{\frac{S}{EJ}}$ **Arrangement:**

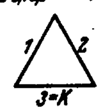


Chart C.- Buckling of a triangular truss within the plane of its system.